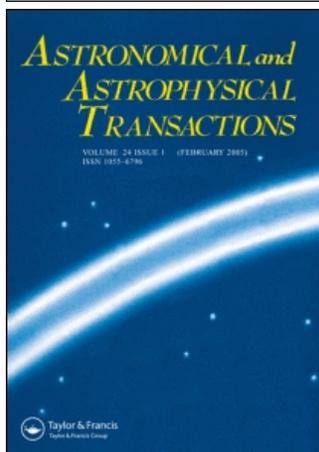


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A. Sapar^a; A. Aret^a

^a Tartu Astrophysical Observatory, Toravere, Estonia

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THE FORMATION OF CHEMICAL PECULIARITIES IN STELLAR ATMOSPHERES

A. SAPAR and A. ARET

Tartu Astrophysical Observatory, Toravere, EE2444 Estonia

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Generalized equations of dynamics for plasma components describing the formation of chemical peculiarities in stellar atmospheres due to gravitational, electrostatic, magnetostatic and radiation fields acting on colliding plasma particles have been derived. The equations describing both diffusion and drift phenomena can be treated intercoupled equations of the Fokker–Planck type. It has been shown that in stellar atmospheres there will be generated the electrostatic field which is important for the separation of light elements and their isotopes. A highly anisotropic diffusion in upper atmospheric layers of magnetic stars has been demonstrated. For the interaction of the plasma components with the radiation field, besides a usual upward directed radiation flux generated acceleration, a more complicated light-induced drift due to the selective resonance-like absorption of radiation by thermally moving plasma particles with definite momentum values is shown to be present. The light induced drift can be directed both upwards or downwards and can be a dominant mechanism in the chemical element separation.

KEY WORDS Stellar atmosphere, chemical peculiarities, diffusion, drift, theory

1 INTRODUCTION

One of essential problems in physics of stars is to explain, in qualitative and quantitative aspects, the formation of chemical peculiarities in stellar atmospheres. As it has been explained from observations and by theory, the chemically peculiar (CP) stars are the stars where no mixing of stellar matter takes place, i.e., the stars without (or with very weak) stellar wind, turbulence and meridional circulation.

Chemical peculiarities of stellar atmospheres have been considered to form in thin upper layers. Two processes of the formation of such anomalies have been discussed. The first one is the accretion of interplanetary dust or of nuclear processed material from supernovae by CP-stars (Havnes and Conti, 1971; Krishna Kumar *et al.*, 1989; Proffitt and Michaud, 1989). However, the concept is insufficiently elaborated and motivated. The second one is a segregation due to diffusion, first proposed by Michaud (1970). In this case, due to different masses, charges and interaction cross-sections of different particles they get different acceleration giving finally the abundance stratification.

The main efforts up to now have been made to explain the observed chemical anomalies of CP-stars as generated mainly by gravity and gradient of radiation pressure (Michaud *et al.*, 1976; Michaud, 1980; Vauclair and Vauclair, 1982; Alecian and Grappin, 1984). In addition, for magnetic Apstars, the action of magnetic field on diffusion processes has been taken into account (Alecian, 1986; Michaud *et al.*, 1981).

It seems to us that for further progress we need possibly general, self-consistent and complex theoretical treatment of the processes of element separation in stellar atmospheres. An attempt to elaborate original equations for such an approach has been made in the present paper. A new phenomenon, the light-induced drift of chemical elements, has been incorporated. The theory of this process was first elaborated by Gelmukhanov and Shalagin (1980). The estimates made thereafter by Atutov and Shalagin (1988) and by Nasyrov and Shalagin (1993) showed its importance for the problems of element diffusion in the atmospheres of chemically peculiar stars. We hope that these equations will be of use both for us in further studies and for other investigators working or interested in the problems of formation of chemical peculiarities in stellar atmospheres.

It is clear that chemical peculiarities can also be of nuclear evolutionary character, say, in stars of spherical population, in eruptive and mass-losing stars. A general picture of CP-stars can be obtained from review papers by Khokhlova (1993), Ryabchikova (1993) and Bisnovatyi-Kogan (1993).

2 COLLISIONAL DIFFUSION OF PLASMA COMPONENTS IN EXTERNAL FIELDS

Let us study the interaction between plasma components i and j . Let the elastically colliding particles to have, after the impact, an isotropic thermalized Maxwell's velocity distribution. Thus, we assume that the indicatrice of scattering is independent of direction. We consider the momentum transfer in such a transition with the cross-section σ_{ji} when particle have number densities n_i and n_j (as a special case, $i = j$). Let the relative particle velocity be v_{ij} , the velocity distribution f_i and f_j , masses m_i and m_j , velocities \vec{v}_i and \vec{v}_j , undisturbed moments $\vec{p}_i = m_i \vec{v}_i$, $\vec{p}_j = m_j \vec{v}_j$ and densities $\rho_i = m_i n_i$, $\rho_j = m_j n_j$.

The particle j collides with particles i with frequency ν_{ji} given by

$$\nu_{ji} = n_i \iint \sigma_{ji}(v_{ij}) v_{ji} f_j f_i d\vec{v}_i d\vec{v}_j. \quad (1)$$

The total frequency of collisions for particles j is thus

$$\nu_j = \sum_i \nu_{ji} \quad (2)$$

and the mean time of free motion

$$t_j = \frac{1}{\nu_j}. \quad (3)$$

The momentum transferred from particles i to particles j in collisions during unit time can be expressed as the force density

$$\dot{\vec{p}}_{ji} = n_j n_i \int \int \vec{p}_i \sigma_{ji}(v_{ij}) v_{ji} f_i f_j d\vec{v}_i d\vec{v}_j. \quad (4)$$

Similarly, we get expression for $\dot{\vec{p}}_{ij}$, exchanging indexes i and j . The mean momentum of colliding particles can be expressed in the form

$$\vec{p}_i = m_i(\vec{v}_i + \vec{a}_i t_i + \vec{V}_i), \quad (5)$$

where the acceleration of particles generated by external forces is denoted by \vec{a}_i and the total velocity of particles due to diffusion and drift, by \vec{V}_i . The external acceleration \vec{a}_i can be generated by gravity, radiation, electrostatic and magnetostatic fields and therefore the resulting process of separation and stratification of chemical elements in stellar atmospheres is due to all of them in different roles.

Taking into account that the mean value of the undisturbed momentum equals to zero, we obtain

$$\vec{p}_{ji} = \lambda_{ij} \rho_i \vec{a}_i + \nu_{ij} \rho_i \vec{V}_i, \quad (6)$$

where the relative impact frequency of particle i with particles j is given by

$$\lambda_{ij} = \frac{\nu_{ij}}{\nu_i},$$

and thus

$$\sum_j \lambda_{ij} = 1.$$

Similarly,

$$\dot{\vec{p}}_{ij} = \lambda_{ji} \rho_j \vec{a}_j + \nu_{ji} \rho_j \vec{V}_j. \quad (7)$$

The total contribution of source terms to plasma component j due to its interaction with plasma components i and caused by external forces can be thus expressed in the form

$$\dot{\vec{p}}_j^+ = \sum_i \dot{\vec{p}}_{ji} = \sum_i \lambda_{ij} \rho_i \vec{a}_i + \sum_i \nu_{ij} \rho_i \vec{V}_i. \quad (8)$$

The total contribution of absorption terms having the similar meaning for momentum loss by particles j in their interaction with other particles can be expressed as the sum where the indices i and j are interchanged, i.e. the roles of donors and acceptors of momentum are interchanged:

$$\dot{\vec{p}}_j^- = \sum_i \dot{\vec{p}}_{ij} = \sum_i \lambda_{ji} \rho_j \vec{a}_j + \sum_i \nu_{ji} \rho_j \vec{V}_j = \rho_j \vec{a}_j + \nu_j \rho_j \vec{V}_j. \quad (9)$$

Taking into account the momentum transfer in interaction with other kinds of particles, we can express the momentum conservation law in gas dynamics for particles j in the form

$$\frac{\partial(\rho_j \vec{V}_j)}{\partial t} + \vec{\nabla} P_j = \rho_j \vec{a}_j + \dot{\vec{p}}_j^+ - \dot{\vec{p}}_j^-, \quad (10)$$

where the partial pressure due to plasma component j is given by

$$P_j = n_j kT.$$

Substituting the expressions found for the interaction terms into Eq. (10), we obtain the final form of the momentum conservation and transfer for plasma components j in external accelerating field in the form

$$\frac{\partial(\rho_j \vec{V}_j)}{\partial t} + \vec{\nabla} P_j = -\nu_j \rho_j \vec{V} + j + \sum_i \lambda_{ij} \rho_i \vec{a}_i + \sum_i \nu_{ij} \rho_i \vec{V}_i. \quad (11)$$

Let us consider now the procedure of finding the diffusion and drift velocity \vec{V}_j in the time independent (stationary) approximation, i.e. omitting the term of time derivative. In this case the linear system of differential equations reduces to a system of linear algebraic in the form

$$-\vec{F}_j = -\nu_j \rho_j \vec{V}_j + \sum_i \nu_{ij} \rho_i \vec{V}_i,$$

or as the condition of equilibrium

$$\nu_j \rho_j \vec{V}_j = \vec{F}_j + \sum_i \nu_{ij} \rho_i \vec{V}_i, \quad (12)$$

where the force density \vec{F}_j denotes expression

$$\vec{F}_j = -\vec{\nabla} P_j + \sum_i \lambda_{ij} \rho_i \vec{a}_i. \quad (13)$$

The quantity \vec{F}_j equals to zero in the case of lacking motion of drift and diffusion. Then this constraint means the condition of hydrostatic equilibrium.

The system of linear equations (12) can be expressed in the form

$$\nu_j \rho_j \vec{V}_j = \sum_k \Gamma_{kj} \vec{F}_k, \quad (12')$$

where the matrix Γ_{kj} can be obtained by the use of the Kramer formulae and it does not depend on velocities \vec{V}_j . It can also be solved in the recurrent way, starting from Eqs. (12) in the form

$$\nu_j \rho_j \vec{V}_j = \vec{F}_j + \sum_i \lambda_{ij} \nu_i \rho_i \vec{V}_i. \quad (14)$$

The quantities λ_{ij} , as relative impact frequencies, are smaller than 1. Thus, we can write a recursive equation for subsequent approximations n and $n+1$ in the form

$$(\nu_j \rho_j \vec{V}_j)_{n+1} = \vec{F}_j + \sum_i \lambda_{ij} (\nu_i \rho_i \vec{V}_i)_n,$$

where

$$(\nu_j \rho_j \vec{V}_j)_0 = \vec{F}_j.$$

Such a procedure means a series expansion with respect to the impact multiplicity, redistributing the momenta originating from external forces \vec{F}_j .

3 THE ELECTROSTATIC FIELD IN PLASMA

Plasma medium in stars must be almost neutral by charge and in the most cases also electric current can be taken equal to zero. In the case of hydrostatic equilibrium, for each plasma component external gravity field and thermal motion of particles yield a smaller density gradient for lighter particles. This circumstance is the primary cause for the separating diffusion and drift of atoms and ions of different chemical elements. Electrons are drastically lighter than the rest of particles and this initiates their diffusion in the direction opposite to the gravity field. However, the smallest relative shift of them generates an electrostatic field, which blocks their escape from the stellar atmosphere, and, speaking figuratively, deforming their stratification due to the external gravity field to correspond to the stratification of ions.

Let us now find the electrostatic field generated in stellar atmosphere.

The condition for the absence of electric current in macromotions can be written in the form

$$\sum_i n_i Z_i \vec{V}_i = 0, \quad (15)$$

where Z_j is the ion charge number. In order to use this constraint for finding the electrostatic field \vec{E} , we multiply both sides of Eq. (12) by $\frac{Z_j e}{\nu_j m_j}$, and thereafter we sum over all kinds of particles. In such a way we obtain, instead of equations describing momentum transfer, the equation from which we can specify the electrostatic field strength. The equation can be written in the form

$$\sum_j \frac{Z_j}{\nu_j m_j} \vec{F}_j + \sum_j \frac{Z_j}{\nu_j m_j} \sum_i \nu_{ij} \rho_i \vec{V}_i = 0. \quad (16)$$

The electric field strength \vec{E} appears explicitly in the equation when we separate, in the total acceleration \vec{a}_i , the contribution due to the electric field, i.e. we take into account that

$$\vec{a}_i = \vec{g}_i + \frac{Z_i e}{m_i} \vec{E}, \quad (17)$$

where the quantity \vec{g}_i includes all external fields excluding electric field. Thus,

$$\vec{F}_j = -\vec{\nabla} P_j + \sum_i \lambda_{ij} \rho_i \left(\vec{g}_i + \frac{Z_i e}{m_i} \vec{E} \right). \quad (18)$$

Substituting the above expression for the external force \vec{F}_j into Eq. (16) and making use of the notation

$$\begin{aligned} \vec{D} &= \sum_j \frac{Z_j}{\nu_j m_j} \left(-\vec{\nabla} P_j + \sum_i \lambda_{ij} \rho_i \vec{g}_i \right), \\ G &= \sum_j \frac{Z_j}{\nu_j m_j} \sum_i \lambda_{ij} \rho_i \frac{Z_i e}{m_i}, \\ \vec{W} &= \sum_i N_i \vec{V}_i, \end{aligned}$$

where

$$N_i = \rho_i \sum_j \frac{Z_j}{m_j} \frac{\nu_{ij}}{\nu_j},$$

we get

$$\vec{D} + G\vec{E} + \vec{W} = 0,$$

and thus the electric field intensity has the form

$$\vec{E} = -\frac{\vec{D} + \vec{W}}{G} = -\frac{\vec{D}}{G} - \sum_i \frac{N_i}{G} \vec{V}_i. \quad (19)$$

Substituting \vec{E} into Eq. (18), we obtain for \vec{V}_j , instead of system of equations (14), a new system from which electric field strength has been eliminated:

$$\nu_j \rho_j \vec{V}_j = \vec{F}'_j + \sum_i \lambda'_{ij} \nu_i \rho_i \vec{V}_i, \quad (20)$$

where

$$\vec{F}'_j = -\vec{\nabla} P_j + \sum_i \lambda_{ij} \rho_i \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right), \quad (21)$$

$$\lambda'_{ij} = \lambda_{ij} - \frac{N_i}{G \nu_i \rho_i} \sum_k \lambda_{kj} \rho_k \frac{Z_k e}{m_k}. \quad (22)$$

The system of equations is of the same form as (14) and only the quantities \vec{F}_j and λ_{ij} are to be merely replaced by \vec{F}'_j and λ'_{ij} . The corresponding solution can be written in the form

$$\nu_j \rho_j \vec{V}_j = \sum_k \Gamma'_{kj} \vec{F}'_k. \quad (23)$$

The new matrix Γ'_{kj} in this equation can be found in the same way as matrix Γ_{kj} in (12).

Eliminating the velocities \vec{V}_j found by (23) from Eq. (19), we have completed the procedure of finding an expression for the electrostatic field strength. The term depending on drift and diffusion velocities \vec{V}_i in this expression means that also the forces generated by particles flows give a small contribution to the formation of electrostatic field.

4 THE DIFFUSION EQUATION FOR THE PLASMA COMPONENTS

The diffusion equation will be derived from the equation of particle conservation for chemical elements. The generalized equation of continuity for particles j in the case of particle transformation can be written in the form

$$\frac{\partial \rho_j}{\partial t} + \vec{\nabla}(\rho_j \vec{V}_j) = \dot{\rho}_j, \quad (24)$$

where the additional term $\dot{\rho}_j$ violating the conservation takes into account the generation and annihilation of particles j in the processes of interaction. Index j can, for instance, specify particles in a given quantum state or as an ion species.

Eliminating quantities $\rho_j \vec{V}_j$ using (23) one obtains

$$\frac{\partial \rho_j}{\partial t} + \vec{\nabla} \left(\sum_k \frac{1}{\nu_j} \Gamma'_{kj} \vec{F}'_k \right) = \dot{\rho}_j. \quad (25)$$

Taking into account Eqs. (20) and (21), we see that the equation obtained is an equation of the Fokker-Planck type describing diffusion and drift motions in gas mixtures for the case when, in addition to buffer gases H and He , there are different interacting admixture gases which can also be transformed between themselves, say, in processes of ionization and excitation, described by the interaction term $\dot{\rho}_j$. If we wish to consider the total diffusion of a given element, the summation over all ions of the given element ϵ must be carried out, i.e. we formulate element densities by sums

$$\rho_\epsilon = \sum_{j \in \epsilon} \rho_j. \quad (26)$$

An important result of such summation is that we get rid of $\dot{\rho}_j$ terms, taking into account that, due to the element conservation law,

$$\sum_{j \in \epsilon} \dot{\rho}_j = 0. \quad (27)$$

We do not need explicitly also quantities ρ_j , because for each element ϵ , making use of degrees of ionization and excitation X_j , we can write

$$X_j = \left. \frac{\rho_j}{\rho_\epsilon} \right|_{j \in \epsilon} = \left. \frac{n_j}{n_\epsilon} \right|_{j \in \epsilon}, \quad (28)$$

where

$$n_\epsilon = \sum_{j \in \epsilon} n_j. \quad (29)$$

From the equation of continuity (24) summing up over plasma ingredients $j \in \epsilon$ and taking into account the element conservation law (27), we obtain

$$\frac{\partial \rho_\epsilon}{\partial t} + \vec{\nabla}(\rho_\epsilon \vec{V}_\epsilon) = 0, \quad (30)$$

where

$$\vec{V}_\epsilon = \frac{1}{\rho_\epsilon} \sum_{j \in \epsilon} \rho_j \vec{V}_j.$$

From Eq. (23) we see, that the momentum of diffusion and drift motion can be written in the form

$$\rho_\epsilon \vec{V}_\epsilon = \sum_{j \in \epsilon} \sum_k \frac{\Gamma'_{kj}}{\nu_j} \vec{F}'_k, \quad (31)$$

where regrouping the terms we can form the total contribution terms $\Gamma'_{\epsilon',\epsilon}$ and the corresponding force vectors $\vec{F}'_{\epsilon'}$, describing interaction between chemical elements. Thus we give to Eq. (31) a new form

$$\rho_{\epsilon} \vec{V}_{\epsilon} = \sum_{\epsilon'} \frac{\Gamma'_{\epsilon',\epsilon}}{\nu_{\epsilon}} \vec{F}'_{\epsilon'}, \quad (32)$$

where the matrix $\Gamma'_{\epsilon',\epsilon}$ and the force vectors $\vec{F}'_{\epsilon'}$ can be found from the following equations:

$$\begin{aligned} \Gamma'_{\epsilon',\epsilon} \vec{F}'_{\epsilon'} &= \nu_{\epsilon} \sum_{j \in \epsilon} \sum_{k \in \epsilon'} \frac{\Gamma'_{kj}}{\nu_j} \vec{F}'_k, \\ \Gamma'_{\epsilon',\epsilon} &= \nu_{\epsilon} \sum_{j \in \epsilon} \sum_{k \in \epsilon'} \frac{\Gamma'_{kj}}{\nu_j}, \\ \nu_{\epsilon} &= \sum_{j \in \epsilon} \nu_j. \end{aligned}$$

Thus, the equation of continuity can finally be written in the form

$$\frac{\partial \rho_{\epsilon}}{\partial t} + \vec{\nabla} \cdot \left(\sum_{\epsilon'} \frac{\Gamma'_{\epsilon',\epsilon}}{\nu_{\epsilon}} \vec{F}'_{\epsilon'} \right) = 0. \quad (33)$$

Wishing to emphasize the Fokker-Planck nature of the equation obtained, we reformulate it starting from Eq. (20):

$$\rho_{\epsilon} \vec{V}_{\epsilon} = \sum_{j \in \epsilon} \frac{1}{\nu_j} \vec{F}'_j + \sum_{j \in \epsilon} \frac{1}{\nu_j} \sum_i \lambda'_{ij} \nu_i \rho_i \vec{V}_i. \quad (34)$$

Let us consider now the first term on the right-hand side of this expression, i.e. the sum

$$\sum_{j \in \epsilon} \frac{\vec{F}'_j}{\nu_j} = \sum_{j \in \epsilon} \frac{1}{\nu_j} \left(-\vec{\nabla} P_j + \sum_i \lambda_{ij} \rho_i \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right) \right). \quad (35)$$

Taking into account that

$$\sum_{j \in \epsilon} \frac{1}{\nu_j} \vec{P}_j = \sum_{j \in \epsilon} \frac{1}{\nu_j} \vec{\nabla} (n_j kT) = \sum_{j \in \epsilon} \frac{X_j}{\nu_j} \vec{\nabla} (n_{\epsilon} kT) + \sum_{j \in \epsilon} \frac{n_{\epsilon} kT}{\nu_j} \vec{\nabla} X_j$$

and

$$\begin{aligned} \sum_{j \in \epsilon} \sum_i \frac{\lambda_{ij}}{\nu_j} \rho_i \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right) &= \sum_{j \in \epsilon} \sum_{\epsilon'} \sum_{i \in \epsilon'} \frac{\lambda_{ij}}{\nu_j} \rho_i \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right) \\ &= \sum_{j \in \epsilon} \sum_{\epsilon'} \sum_{i \in \epsilon'} \frac{\lambda_{ij}}{\nu_j} X_i \rho_{\epsilon'} \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right) \\ &= \sum_{\epsilon'} \sum_{i \in \epsilon'} \rho_{\epsilon'} \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right) \sum_{j \in \epsilon} \frac{\lambda_{ij}}{\nu_j} X_i \end{aligned}$$

and denoting

$$\begin{aligned}
 P_\epsilon &= n_\epsilon kT, \\
 \vec{Y}_\epsilon &= \sum_{j \in \epsilon} \frac{\vec{\nabla} X_j}{\nu_j}, \\
 \frac{1}{\nu_\epsilon^*} &= \sum_{j \in \epsilon} \frac{X_j}{\nu_j}, \\
 \vec{F}_\epsilon^* &= \nu_\epsilon^* \sum_{j \in \epsilon} \frac{\vec{F}_j'}{\nu_j}, \\
 T_{i\epsilon} &= \sum_{j \in \epsilon} \frac{\lambda_{ij}}{\nu_j} X_i,
 \end{aligned}$$

we find

$$\frac{\vec{F}_\epsilon^*}{\nu_\epsilon^*} = -\frac{\vec{\nabla} P_\epsilon}{\nu_\epsilon^*} - P_\epsilon \vec{Y}_\epsilon + \sum_{\epsilon'} \sum_{i \in \epsilon'} T_{i\epsilon} \rho_{\epsilon'} \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right). \quad (36)$$

Now we define a new matrix $A_{\epsilon'\epsilon}$ by

$$A_{\epsilon'\epsilon} = \sum_{i \in \epsilon'} T_{i\epsilon}$$

and the effective acceleration vector $\vec{a}_{\epsilon'}$ by

$$A_{\epsilon'\epsilon} \vec{a}_{\epsilon'} = \sum_{i \in \epsilon'} T_{i\epsilon} \left(\vec{g}_i - \frac{Z_i e}{m_i} \frac{\vec{D}}{G} \right).$$

Thus, we can write an expression for the effective force \vec{F}_ϵ^* , which affects the element ϵ , in the form

$$\frac{\vec{F}_\epsilon^*}{\nu_\epsilon^*} = -\frac{\vec{\nabla} P_\epsilon}{\nu_\epsilon^*} - P_\epsilon \vec{Y}_\epsilon + \sum_{\epsilon'} A_{\epsilon'\epsilon} \rho_{\epsilon'} \vec{a}_{\epsilon'}. \quad (37)$$

Consider the second term in expression (34), i.e. the sum

$$\begin{aligned}
 \rho_\epsilon \vec{V}'_\epsilon &\equiv \sum_{j \in \epsilon} \frac{1}{\nu_j} \sum_i \lambda'_{ij} \nu_i \rho_i \vec{V}_i \\
 &= \sum_{\epsilon'} \sum_{j \in \epsilon} \sum_{i \in \epsilon'} \frac{1}{\nu_j} \lambda'_{ij} \nu_i \rho_i \vec{V}_i.
 \end{aligned} \quad (38)$$

Now, by the use of new coefficients $\lambda'_{i\epsilon}$ defined by

$$\lambda'_{i\epsilon} \nu_\epsilon \sum_{j \in \epsilon} \frac{\lambda'_{ij}}{\nu_j},$$

we can write expression (38) in the form

$$\rho_\epsilon \vec{V}'_\epsilon = \frac{1}{\nu_\epsilon} \sum_{\epsilon'} \sum_{i \in \epsilon'} \lambda'_{i\epsilon} \nu_i \rho_i \vec{V}_i. \quad (39)$$

Denoting

$$\begin{aligned} \Lambda_{\epsilon'\epsilon} &= \sum_{i \in \epsilon'} \lambda'_{i\epsilon}, \\ \Lambda_{\epsilon'\epsilon} \vec{V}'_{\epsilon'} &= \frac{1}{\nu_{\epsilon'} \rho_{\epsilon'}} \sum_{i \in \epsilon'} \lambda'_{i\epsilon} \nu_i \rho_i \vec{V}_i, \end{aligned} \quad (40)$$

we find finally

$$\rho_\epsilon \vec{V}'_\epsilon = \sum_{\epsilon'} \frac{\Lambda_{\epsilon'\epsilon}}{\nu_\epsilon} \nu_{\epsilon'} \rho_{\epsilon'} \vec{V}'_{\epsilon'}. \quad (41)$$

Making use of expressions (34), (36) and (41) in the equation of continuity (30), we obtain

$$\frac{\partial \rho_\epsilon}{\partial t} + \vec{\nabla} \cdot \left(\frac{\vec{F}'_\epsilon}{\nu'_\epsilon} + \sum_{\epsilon'} \frac{\Lambda_{\epsilon'\epsilon}}{\nu_\epsilon} \nu_{\epsilon'} \rho_{\epsilon'} \vec{V}'_{\epsilon'} \right) = 0 \quad (42)$$

or

$$\frac{\partial \rho_\epsilon}{\partial t} + \vec{\nabla} \cdot \left(\frac{\vec{F}'_\epsilon}{\nu'_\epsilon} + \sum_{\epsilon'} \Lambda_{\epsilon'\epsilon}^* \rho_{\epsilon'} \vec{V}'_{\epsilon'} \right) = 0, \quad (43)$$

where

$$\Lambda_{\epsilon'\epsilon}^* = \frac{\nu_{\epsilon'}}{\nu_\epsilon} \Lambda_{\epsilon'\epsilon}.$$

Substituting Eq. (37) into Eq. (43), we obtain the equation of continuity in the form from which we see that the equation can be treated as a generalized Fokker-Planck type diffusion equation, or more exactly as a generalized Fokker-Planck type system of equations for plasma components.

Electrons were treated in the systems of equations (14), (20) and (24) as a particular element which can be created and annihilated in ionization and recombination processes, respectively. Thus, in contrast to the condition $\dot{\rho}_e = 0$ holding for chemical elements, for electrons we have $\dot{\rho}_e \neq 0$. However, we do not need to specify the number density of electrons from the equation of continuity, but it can be found from the condition of electroneutrality for the plasma:

$$\sum_j Z_j n_j = 0,$$

or

$$\sum_\epsilon Z_\epsilon n_\epsilon = 0,$$

where Z_ϵ is the mean degree of ionization for element ϵ . The effective charge number for electrons is $Z_e = -1$.

Similarly, from constraint (15), meaning the absence of electric currents on a macroscale, we can specify the electron drift velocity \vec{V}_e .

In Eq. (24), the quantity $\dot{\rho}_j$ is responsible for the transformations of particles of type j in their interaction with other particles. As typical representatives of such processes of transformation, are excitation, ionization and recombination due to inelastic particle impacts and their interaction with photons. The processes of transformation specify the thermodynamical state of plasma and, ignoring very slow motion and temporal changes, the generalized equation of continuity (25) reduces to the equations of statistical equilibrium (the equation of stationarity)

$$\dot{\rho}_j = 0,$$

which means that the interaction processes populating and depopulating a particle state j are balances. In the equations of statistical equilibrium, the detailed thermodynamical equilibrium in stellar atmospheres is violated by a non-Planckian radiation field, giving rise to deviation from LTE in the populations of atomic states.

5 PLASMA INTERACTION WITH THE PHOTON FIELD IN BOUND-BOUND ELECTRON TRANSITIONS

Up to now we have studied the impacts between particles neglecting their interaction with photons, i.e. with radiation field.

Let an energetically lower state of atomic particle be l and the higher (upper) state u . In photon adsorption by particles in state l , the energy and momentum of the photon will be transferred to particle in state u . The opposite momentum transfer from state u to state l happens in spontaneous radiative transitions.

In the result of the photon absorption, the radiation flux acts on particles in state u in unit volume with force

$$\vec{f}_{ul} = \frac{\pi}{c} \int n_l \sigma_{ul}(\nu) \vec{F}_\nu d\nu. \quad (44)$$

In this expression, $\pi \vec{F}_\nu$ is the monochromatic flux (erg/cm² s Hz), n_l is the number density of particles in the state l and $\sigma_{ul}(\nu)$ is the cross-section of the photon absorption at frequency ν in the transition $l \rightarrow u$, which can be expressed in the form

$$\sigma_{ul}(\nu) = \sigma_{ul}^0 W(u_\nu, a), \quad (45)$$

where σ_{ul}^0 denotes the photon absorption cross-section in the transition $l \rightarrow u$, which can be expressed by the use of the corresponding oscillator strength f_{ul} in the form

$$\sigma_{ul}^0 = \frac{\pi e^2 f_{ul}}{m_e c \Delta\nu_D} \quad (46)$$

where $\Delta\nu_D$ is the Doppler width of the spectral line.

The normalized frequency distribution ($\int W(u_\nu, a) du_\nu = 1$) or the profile function in a spectral line has the form

$$W(u_\nu, a) = \frac{a}{\pi^{3/2}} \int_{-\infty}^{\infty} \frac{e^{-y^2}}{(u_\nu - y)^2 + a^2} dy = \int_{-\infty}^{\infty} W(u_\nu, a, y) dy, \quad (47)$$

called the Voigt function. The parameter a in the Voigt function is the ratio of the characteristic widths of the Lorentz and Doppler profiles:

$$a = \frac{\Delta\nu_L}{\Delta\nu_D},$$

the dimensionless frequency parameter u_ν in the Voigt function is defined by

$$u_\nu = \frac{\nu - \nu_0}{\Delta\nu_D}, \quad (48)$$

and the dimensionless momentum as integration argument is given by the expression

$$y = \frac{M_\nu}{\sqrt{2MkT}}, \quad (49)$$

where M is the mass of the absorbing particle.

In the electron transition $l \rightarrow u$, the momentum of the atomic particle is transferred from the particle in the state l to the particle in the state u . The momentum transferred in this way in unit volume during unit time interval has the same form as in the case of impact of two atomic particles (Eq. (4)), being proportional to the volume densities of inelastically colliding particles n_l and n_ν and with absorption cross-section while particles must have resonance velocity, determined by values of parameter y in the Voigt profile function. Namely, considering the momentum of atomic particles transferred from the state l to the state u , the velocity component of the atomic particle in the flight direction of the photon to be absorbed given by Eq. (49) must fit to the photon frequency, specified by values of parameters u_ν . The absorption cross-section in the case of a fixed value of u_ν is $\sigma_{ul}^0 W(u_\nu, a, y)$ and the momentum will be transferred just proportionally to this cross-section values and in the direction of the photon flight. According to these explications, we can write

$$\vec{f}_{ul}^D = \sqrt{2MkT} \iint n_l \sigma_{ul}^0 y W(u_\nu, a, y) \vec{i} n_\nu dy dn_\nu d\Omega, \quad (50)$$

where \vec{i} is the unit vector in the direction of the photon flight and Ω is the solid angle. The expression obtained describes the transfer of momenta of the atomic particles in the lower quantum state to the atomic particles in the upper quantum state in the photon absorption process. The momentum transfer in the manner described above is an additional effect to the momentum transfer of photons themselves, which manifests itself as the force generated by the radiation flux πF_ν . As it has been demonstrated by Nasyrov and Shalagin (1993), this additional effect, called by

them the light-induced drift, can be very important in the formation of separation of chemical elements in stellar atmospheres. As an indirect testimony of that matter serves the fact that in photon absorption it holds $\frac{Mv^2}{2} = h\nu$ or $\frac{h\nu}{c} = \frac{v}{2c}Mv$. This shows that the photon momenta are essentially smaller than the momenta of atomic particles.

Photon number density for unit volume in a given direction is connected with its monochromatic intensity I_ν by

$$n_\nu = \frac{I_\nu}{h\nu}, \quad (51)$$

and thus

$$\frac{\pi \vec{F}_\nu}{h\nu} = \int_{(4\pi)} c \vec{i} n_\nu d\Omega.$$

Now we shall give a more convenient form to Eq. (50). Therefore we introduce a new variable

$$z_\nu = u_\nu - y. \quad (52)$$

Replacing variable y by variable z_ν we get the Voigt function in the form

$$W(u_\nu, a) = \frac{a}{\pi^{3/2}} \int_{-\infty}^{\infty} \frac{e^{-(u_\nu - z_\nu)^2}}{z_\nu^2 + a^2} dz_\nu. \quad (53)$$

From here we see that

$$\frac{\partial W(u_\nu, a)}{\partial u_\nu} = -\frac{2a}{\pi^{3/2}} \int_{-\infty}^{\infty} (u_\nu - z_\nu) \frac{e^{-(u_\nu - z_\nu)^2}}{z_\nu^2 + a^2} dz_\nu. \quad (54)$$

Taking into account that $y = u_\nu - z_\nu$, we can write

$$\vec{f}_{ul}^D = -2\sqrt{2MkT} \int_0^\infty n_l \sigma_{ul}^0 \frac{\partial W(u_\nu, a)}{\partial u_\nu} \frac{\pi \vec{F}_\nu}{ch\nu} d\nu. \quad (55)$$

Integrating by parts and taking into account that the boundary values of the Voigt function at infinity tend to zero, we obtain

$$\vec{f}_{ul}^D = 2\sqrt{2MkT} \int_0^\infty n_l \sigma_{ul}^0 W(u_\nu, a) \frac{\pi}{ch\nu} \frac{\partial \vec{F}_\nu}{\partial u_\nu} d\nu. \quad (56)$$

This interesting result shows that the directed momentum transfer in spectral lines depends on the distribution of the spectral gradient of the radiation flux in the spectral line frequencies. Thus, accounting for the interaction with photons, we obtain additional terms for momentum transfer to the quantum state u in the form

$$\vec{p}_u = \sum_{l < u} \left(\vec{f}_{ul}^{\vec{r}} + \vec{f}_{ul}^D - A_{ul} \vec{p}_u n_u \right), \quad (57)$$

where

$$\vec{p}_u = M\vec{V}_u.$$

The last term in expression (57) takes into account spontaneous transitions $u \rightarrow l$, accompanied by the corresponding decrease of the momentum \vec{p}_u .

The two last terms work upon at the quantum state l in opposite direction than upon the state u in Eq. (57), and the term of photon pressure force \vec{f}_{ul}^r does not affect it. Thus we can write

$$\dot{\vec{p}}_l = \sum_{u>l} \left(-\vec{f}_{ul}^D + A_{ul}\vec{p}_u n_u \right). \quad (58)$$

The quantities A_{ul} and σ_{ul}^0 are interrelated by

$$\sigma_{ul}^0 = \frac{c^2}{\nu^2 \Delta\nu_D} \frac{g_u}{g_l} \frac{A_{ul}}{8\pi}. \quad (59)$$

According to above notation,

$$\begin{aligned} n_u \vec{p}_u &= \rho_u \vec{V}_u, \\ n_l \vec{p}_l &= \rho_l \vec{V}_l, \end{aligned}$$

Introducing for momentum transfer the expressions free from transition probabilities to be found by

$$\begin{aligned} \vec{j}_{ul}^D &= \frac{\vec{f}_{ul}^D}{A_{ul}}, \\ \vec{j}_{ul}^r &= \frac{\vec{f}_{ul}^r}{A_{ul}}, \end{aligned} \quad (60)$$

the additional terms describing the interaction of the photon field with atomic particles in the case of bound-bound electron transitions can be expressed in the form

$$\dot{\vec{p}}_u^+ = \sum_{l<u} A_{ul} \left(\vec{j}_{ul}^D + \vec{j}_{ul}^r - \rho_u \vec{V}_u \right), \quad (61)$$

$$\dot{\vec{p}}_l^- = - \sum_{u>l} A_{ul} \left(\vec{j}_{ul}^D - \rho_u \vec{V}_u \right). \quad (62)$$

Now we can add the terms (61) and (62) into the main equations of the momentum transfer. The additional terms, which take into account the interaction of the state j with lower bound states l and with higher bound states u , can be written in the form

$$\begin{aligned} \dot{\vec{p}}_j^b &= \dot{\vec{p}}_j^+ + \dot{\vec{p}}_j^- \\ &= \sum_{l<j; l, j \in s} A_{jl} \left(\vec{j}_{jl}^D + \vec{j}_{jl}^r - \rho_j \vec{V}_j \right) - \sum_{u>j; u, j \in s} A_{uj} \left(\vec{j}_{uj}^D - \rho_u \vec{V}_u \right), \end{aligned} \quad (63)$$

where by s is marked the ion type.

Thus, we have specified the additional terms, which describe momentum transfer to plasma components in their interaction with photons in bound-bound electron transitions.

6 PLASMA INTERACTION WITH THE PHOTON FIELD IN ELECTRON BOUND-FREE AND FREE-FREE INTERACTIONS

It is clear that, in electron bound-free and free-free transitions, the energy and momentum will be transferred partially to created ion in the ground state and partially to electron getting free or getting additional momentum.

Let us study now how much of momentum will be transferred to forming ion and how much to electron. The conservation laws of momentum and energy can be expressed here in the form

$$h\vec{k} + M\vec{v}_A^0 + m_e\vec{v}_e^0 = M\vec{v}_A + m_e\vec{v}_e, \quad (64)$$

$$h\nu + \frac{M(v_A^0)^2}{2} + \frac{m_e(v_e^0)^2}{2} = E_s + \frac{Mv_A^2}{2} + \frac{m_ev_e^2}{2}, \quad (65)$$

where E_s is the binding energy absorbed by the ion, \vec{k} is the photon wave number, M and v_A are respectively the mass and velocity of the atomic particles, m_e and \vec{v}_e are respectively the electron mass and velocity.

By index zero are marked the pre-impact values of corresponding quantities. A part of photon energy, namely

$$h\Delta\nu = h\nu - E_s, \quad (66)$$

will be transformed into the kinetic energy.

Let us consider now the bound-free transitions. For them we have a constrain $\vec{v}_A^0 = \vec{v}_e^0$ and E_s is the ionization energy from the initial state. Making use of a coordinate system co-moving with the ion prior to impact, i.e. taking $\vec{v}_A^0 = 0$, we obtain

$$h\Delta\nu = \frac{Mv_A^2}{2} + \frac{m_ev_e^2}{2}, \quad (67)$$

$$h^2k^2 = M^2v_A^2 + 2Mm_e\mu v_A v_e + m_e^2v_e^2, \quad (68)$$

where

$$\mu = \frac{\vec{v}_A \vec{v}_e}{v_A v_e}.$$

Introducing a new parameter β by

$$hk = Mv_A + \beta m_e v_e, \quad \beta \in [-1, +1], \quad (69)$$

taking into account that $Mv_A \geq 0$ and $m_e v_e \geq 0$, and substituting the expression for electron kinetic energy from Eq. (67) into Eq. (69), we obtain

$$(hk - Mv_A)^2 = 2m_e\beta^2 \left(h\Delta\nu - \frac{Mv_A^2}{2} \right), \quad (70)$$

and hence

$$Mv_A = \frac{1}{\alpha} \left(hk \pm \sqrt{(hk)^2 - \alpha(h^2k^2 - 2m_e\beta^2 h\Delta\nu)} \right),$$

where

$$a = 1 + \frac{\beta^2 m_e}{M}.$$

Taking into account that $m/\mu < 1/1840$ and $|\beta| \leq 1$, we shall use further only the approximate value $a = 1$. In this approximation,

$$Mv_A = hk \pm \beta\sqrt{2meh\Delta\nu}, \quad (71)$$

and thus, based on Eq. (69) and taking into account that $m_e v_e \geq 0$, we obtain

$$m_e v_e = \sqrt{2m_e h \Delta\nu}, \quad (72)$$

or

$$\frac{1}{2}m_e v_e^2 = h\Delta\nu.$$

This means that, in the bound-free transitions, practically whole additional energy is transferred to electrons.

Solving Eq. (68) for Mv_A , we find

$$Mv_A = -\mu m_e v_e \pm \sqrt{h^2 k^2 - m_e^2 v_e^2 (1 - \mu^2)}. \quad (73)$$

Substituting Mv_A from Eq. (69) into this expression and solving it for β , we obtain

$$\begin{aligned} \beta &= \frac{hk}{m_e v_e} + \mu - \sqrt{\left(\frac{hk}{m_e v_e}\right)^2 + \mu^2 - 1} \\ &= \frac{hk}{\sqrt{2m_e h \Delta\nu}} + \mu - \sqrt{\frac{hk^2}{2m_e \Delta\nu} + \mu^2 - 1}. \end{aligned} \quad (74)$$

where the right solution of the quadratic equation is found from limiting cases (if $\mu = 1$, then $\beta = 1$ and if $\mu = -1$, then $\beta = -1$). This expression shows connection between parameters β and μ .

Whereas the energy of a liberated electron equals to $h\Delta\nu$, its momentum is given by

$$m_e \vec{v}_e = \vec{i}_e \frac{2h\Delta\nu}{v_e}, \quad (75)$$

where \vec{i}_e is a unit vector in the direction of the electron motion. Let \vec{i} be a unit vector directed along the photon momentum vector. In these notations we can write the momentum conservation law in the form

$$\frac{h\nu}{c} \vec{i} = M\vec{v}_A + \frac{2h\Delta\nu}{v_e} \vec{i}_e. \quad (76)$$

From this expression we can find the momentum values transferred from photons to atoms and electrons. However, in the context of the present study, we are not

interested in momentum transfer to atomic particles in an individual interaction process, but only in the total statistical transfer rates.

The total momentum transferred to electrons equals to zero, because integrating over electrons and taking into account that the unit vector \vec{i}_e is, relative to \vec{v}_e , an odd- power function while $F(\vec{v}_e)$ is an even-power function, we obtain

$$\int m_e \vec{v}_e f(\vec{v}_e) d\vec{v}_e = 2h\Delta\nu \int \frac{\vec{i}_e}{v_e} f(\vec{v}_e) d\vec{v}_e = 0. \quad (77)$$

Therefore, it is clear that the total momentum of the photon field must be transferred to atomic particles. The momentum transferred from photons to atomic particles per unit volume and unit time interval can be expressed in the form

$$\dot{\vec{p}}_s^J = \frac{\pi}{c} \int k_\nu^{s'} \vec{F}_\nu d\nu, \quad (78)$$

The quantity $k_\nu^{s'}$ is the radiation absorption coefficient due to the bound-free transitions $s' \rightarrow s$. This momentum, as we have seen, is transferred to generated ions s in the ground state G .

Electrons in the free-free transitions can be considered as connected with ions and thus the photon absorption by them is accompanied by electron jump from one hyperbolic orbit to another. This circumstance makes us clear that we can use the same formulae as in the case of electron bound-free transitions. The only difference is that now the binding energy $E_s = 0$, and thus $h\Delta\nu = h\nu$. Consequently, it follows that in electron free-free transition practically whole photon momentum is transferred to ions and the energy is transferred to electrons. Thus, for free-free transitions we can express the momentum transferred from photons to absorbing ions s per unit volume and unit time interval in the same form as in the case of bound-free transitions

$$\dot{\vec{p}}_s^J = \frac{\pi}{c} \int k_\nu^{Jf,s} \vec{F}_\nu d\nu, \quad (79)$$

with the only difference that here the absorption coefficient $k_\nu^{Jf,s}$ corresponds to free-free electron transitions of ion s .

The expression

$$\dot{\vec{p}}_{sG} = \dot{\vec{p}}_s^J + \dot{\vec{p}}_s^f \quad (80)$$

describes the total continuous absorption, the final state of which is ion s in the ground state G , to which the whole momentum of the absorbed photons is transferred.

The free electrons are subject to a force originating from Thomson scattering of photons by electrons. The corresponding momentum transferred to plasma by the medium of electrons is given by

$$\dot{\vec{p}}_s^f = \frac{\pi}{c} \sigma_0 n_e \int \vec{F}_\nu d\nu, \quad (81)$$

where σ_0 is the Thomson cross-section of photon scattering by free electrons.

Summing up the results found for the interaction of photons with plasma particles, we can write for the additional force \vec{f}_j^* an expression which must be added to the momentum transfer equation (20) for the plasma components

$$\vec{f}_j^* = \dot{\vec{p}}_j^{bb} + \dot{\vec{p}}_{sG} + \dot{\vec{p}}^f. \quad (82)$$

The first term (Eq. (63)) in this formula gives the contribution to the excited states, the second (Eq. (78)–(80)) gives the contribution to the ground states and the last term (Eq. (81)), to free electrons.

Thus, we have derived the final expression for the force acting on plasma components due to the radiative flux.

In order to describe also the bound–bound transitions by the use of system (12) of linear equations relative to \vec{V}_j , we must combine the terms of spontaneous transitions involving velocities \vec{V}_j in Eq. (63) with the particle collision terms in Eq. (12). To do so, we should treat spontaneous transitions as additional impacts, meaning that collision frequencies ν_{ij} in Eq. (12) are to be replaced by

$$\nu_{ij}^* = \nu_{ij} + A_{ji}\Theta_{ji}, \quad (83)$$

whereas $A_{ji}\Theta_{ji}$ differs from zero only in the case of bound–bound transitions and

$$\Theta_{ji} = \begin{cases} 1, & \text{if } j > i \\ 0, & \text{if } j \leq i. \end{cases}$$

Thus, further routine in derivation of generalized equations which contain quantities ν_{ji}^* will conserve the same steps as in the discussion above. The only modification following from (83) is that the impact frequencies ν_j should be replaced by

$$\nu_j^* = \nu_j + A_j,$$

where

$$A_j = \sum_{i \in s, i < j} A_{ji}.$$

In the calculation of the accelerations of different origins treated above, the most complicated problem is to specify the radiative acceleration, especially the one due to bound–bound electron transitions. As a new effect, there appear the momentum transfer from particles of a given bound state to particles in another bound state due to the photon absorption, the so-called light-induced drift of particles, embodied by quantities \vec{j}_{ul}^D in Eq. (63). This phenomenon was studied theoretically and experimentally by Gelmukhanov and Shalagin (1980), Atutov and Shalagin (1988) and by Nasyrov and Shalagin (1993). The nature of this phenomenon can be illustrated by a simple example.

Consider a particle with two bound states, one of which is the ground state G and another is the excited state E . The probability (frequency) of electron transition $E \rightarrow G$ denoted by A_{EG} . Further, we assume that there are no external force except one due to the radiation transfer in the spectral line. Let the pressure of particles

both in the ground state and in the excited state have non-zero gradients. In this case our system of equations (12) generalized by (83) reduces to a very simple form:

$$\begin{aligned}\vec{\nabla} P_E &= -A_{EG}\rho_E\vec{V}_E - \nu_E\rho_E\vec{V}_E + A_{EG}\vec{j}_{EG}^D, \\ \vec{\nabla} P_G &= A_{EG}\rho_E\vec{V}_E - \nu_G\rho_G\vec{V}_G - A_{EG}\vec{j}_{EG}^D,\end{aligned}$$

from where we find

$$\begin{aligned}\rho_E\vec{V}_E &= \frac{A_{EG}\vec{j}_{EG}^D - \vec{\nabla} P_E}{A_{EG} + \nu_E}, \\ \rho_G\vec{V}_G &= -\frac{1}{\nu_G} \left(\vec{\nabla} P_G + A_{EG}\vec{j}_{EG}^D \right) - \frac{E_{EG} \left(\vec{\nabla} P_E - A_{EG}\vec{j}_{EG}^D \right)}{\nu_G(A_{EG} + \nu_E)}.\end{aligned}$$

Thus, in such a case the light-induced drift of particles is non-zero, namely

$$\begin{aligned}\rho\vec{V} &= \rho_E\vec{V}_E + \rho_G\vec{V}_G \\ &= -\frac{1}{\nu_G} \left(\vec{\nabla} P_G + A_{EG}\vec{j}_{EG}^D \right) \\ &\quad - \frac{1}{A_{EG} + \nu_E} \left(1 + \frac{A_{EG}}{\nu_G} \right) \left(\vec{\nabla} P_E - A_{EG}\vec{j}_{EG}^D \right).\end{aligned}$$

This example shows that the additional light-induced forces $A_{EG}\vec{j}_{EG}^D$ play the same role for the ground state as the gas pressure gradient but they act in the opposite direction for the excited state. Calculations show that the light-induced drift can by some decimal orders exceed the diffusion terms, containing gradients $\vec{\nabla} P_G$ and $\vec{\nabla} P_E$ (Nasyrov and Shalagin, 1993).

With this example we accomplish the treatment of the interaction between photons and plasma components. To get realistic drift velocity values for chemical elements is a complicated task because the value and direction of the light-induced drift depend on mutual blending of spectral lines of a given element, its isotopes and spectral lines of different elements. In addition to such a complication, the spectral lines shift in magnetic fields due to the Zeeman effect and thus the role of bound-bound electron transitions can essentially depend upon the stellar magnetic field. The magnetic field also has a direct influence on the dynamics of plasma components. The next section is devoted to this problem.

7 MAGNETIC FIELD ACTION ON THE DRIFT AND DIFFUSION OF PLASMA COMPONENTS

Many chemically peculiar stars have strong magnetic fields reaching 10^3 – 10^4 G. In such a magnetic field, the charged particles, electrons and ions, are subject to force which exceed gravitational and electrostatic forces by many orders of magnitude.

Thus, the proton acceleration in magnetic field 10^3 G, in its motion perpendicular to \vec{H} , at temperature 10^4 K, with the mean energy $\frac{3}{2}kT$ is

$$\frac{ev_p H}{cM_p} = \frac{e}{c} \sqrt{\frac{kT}{M_p^3}} H \approx 10^{13} \text{ cm/s}^2,$$

and the acceleration of electrons under such conditions due to their small masses is even 10^{16} cm/s². These quantities exceed drastically a typical gravity 10^4 cm/s².

Averaged motion of particles is not at all governed by magnetic field alone. Strong magnetic field generates large centripetal force but time-averaged acceleration of charged particles is generated only by magnetic field changes in strength and direction.

Let us consider how magnetic field acts on the motion of plasma particles. First of all we shall discuss the motion of isolated impact-free charged plasma particle in the magnetic field. The equation of motion for such a particle can be expressed by

$$\frac{d\vec{p}}{dt} = Ze \left(\vec{E} + [\vec{v} \times \vec{H}] \right), \quad (84)$$

where \vec{E} is the electric fields strength and \vec{H} is the magnetic field strength. In approximation $\vec{E} = 0$ we obtain

$$\frac{d\vec{p}}{dt} = Ze [\vec{v} \times \vec{H}], \quad (85)$$

and from Maxwell's equations it follows that, in this approximation,

$$\frac{\partial \vec{H}}{\partial t} = 0. \quad (86)$$

Further, we suppose that within the circle drawn by the Larmor radius the magnetic field can be considered as constant. In order to formulate this condition mathematically, we define the characteristic length L by

$$\frac{1}{L} = \max \left| \frac{1}{H} \frac{\partial H_q}{\partial r_k} \right|, \quad (87)$$

where q and k are the coordinate indices. Denoting the Larmor period of the particle by

$$\tau = \frac{2\pi}{\omega_L} = \frac{2\pi m}{ZeH}, \quad (88)$$

we can write the condition of a slow change of the magnetic field strength in the form

$$L \gg v\tau. \quad (89)$$

Further, we shall restrict ourselves to the first order approximation in vr/L . The error of such approximation does not exceed essentially $(vr/L)^2$. Such an approximation is well-known as Alfvén's approximation (Alfvén and Fälthammar, 1963).

Particle motion is described much simpler using the concept of the leading centre, defined as the interaction point G of the momentary axis of revolution with the plane perpendicular to \vec{H} on which the revolving particle is situated. The distance of the point G from the revolving particle is given by the Larmor radius

$$R_L = \frac{mv \sin \alpha}{ZeH},$$

where α is the angle between the particles velocity \vec{v} and the magnetic field strength \vec{H} .

In the case of uniform time-independent magnetic field, we can consider particle motion as a superposition of a rotation in the plane perpendicular to the magnetic field and a uniform rectilinear motion along magnetic field lines. If the components of the magnetic field strength gradient satisfy condition (89), then magnetic field acting on the particle equals to its value at the leading centre. Thus, also in the case of a small gradient of the magnetic field strength, we can use the concept of the local leading centre which gives essential simplification of the problem, because instead of a spiral-like motion we can consider a simpler motion of the leading centre, characterized by quantities averaged over the Larmor period. As such averaged quantities, we use the drift velocity perpendicular to the magnetic field line \vec{v}^\perp expressed by (Rossi and Olbert, 1974)

$$\vec{v}^\perp = \frac{mv^2 c}{ZeH} \left\{ \frac{1}{2} \sin^2 \alpha \frac{[\vec{H} \times \vec{\nabla} H]}{H^2} + \cos^2 \alpha \frac{[\vec{H} \times (\vec{H} \vec{\nabla}) \vec{H}]}{H^3} \right\}, \quad (90)$$

and the acceleration parallel to the magnetic field line,

$$\frac{d\vec{v}^\parallel}{dt} = -\frac{\vec{\mu}_L}{m} (\vec{i}_H \vec{\nabla}) H, \quad (91)$$

where \vec{i}_H is the unit vector in the direction of \vec{H} and μ_L is the magnetic moment, generated by a revolving charged particle

$$\vec{\mu}_L = \frac{mv^2 \sin^2 \alpha}{2H} \vec{i}_H = \frac{W^\perp}{H} \vec{i}_H, \quad (92)$$

where W^\perp is the component of kinetic energy W perpendicular to the magnetic field strength, i.e. $W^\perp = W \sin^2 \alpha$, where $W = \frac{1}{2} mv^2$.

Further, we restrict ourselves to a dipole magnetic field with magnetic moment $\vec{\mu}$ localized at the stellar centre, thus taking

$$\vec{H} = \frac{3(\vec{\mu} \vec{r}) - \vec{\mu} r^2}{r^5}. \quad (93)$$

Making use of the magnetic spherical coordinates r, ϑ, φ (Figure 1) and of the corresponding unit basis vectors $\vec{i}_r, \vec{i}_\vartheta, \vec{i}_\varphi$, we can express the dipole magnetic field strength in the form

$$\vec{H} = \frac{\mu}{r^3} (2 \cos \vartheta \vec{i}_r + \sin \vartheta \vec{i}_\vartheta). \quad (94)$$

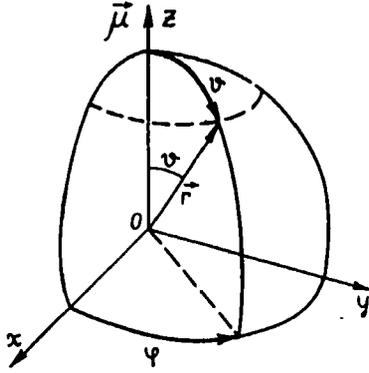


Figure 1 Magnetic spherical co-ordinates.

From here it follows that

$$H = \frac{\mu}{r^3} \sqrt{3 \cos^2 \vartheta + 1}, \quad (95)$$

and thus the unit vector \vec{i}_H in the direction of the magnetic field can be written as

$$\vec{i}_H = \frac{\vec{H}}{H} = \frac{2 \cos \vartheta \vec{i}_r + \sin \vartheta \vec{i}_\vartheta}{\sqrt{3 \cos^2 \vartheta + 1}}. \quad (96)$$

In order to find the drift velocity of the leading centre \vec{v}^\perp and the force along the magnetic lines $F^\parallel = m \frac{d\vec{v}^\parallel}{dt}$ for the dipole magnetic field, we start by making some auxiliary calculations.

In spherical coordinates, the operator $\vec{\nabla}$ has the form

$$\vec{\nabla} = \vec{i}_r \frac{\partial}{\partial r} + \vec{i}_\vartheta \frac{1}{r} \frac{\partial}{\partial \vartheta} + \vec{i}_\varphi \frac{1}{r \sin \vartheta} \frac{\partial}{\partial \varphi}. \quad (97)$$

Thus, for the operator $(\vec{i}_H \cdot \vec{\nabla})$ we find

$$(\vec{i}_H \cdot \vec{\nabla}) = \frac{2 \cos \vartheta}{\sqrt{3 \cos^2 \vartheta + 1}} \frac{\partial}{\partial r} + \frac{\sin \vartheta}{r \sqrt{3 \cos^2 \vartheta + 1}} \frac{\partial}{\partial \vartheta}. \quad (98)$$

By the direct calculations we obtain

$$(\vec{i}_H \cdot \vec{\nabla}) H = -\frac{3\mu \cos \vartheta}{r^4} \frac{5 \cos^2 \vartheta + 3}{3 \cos^2 \vartheta + 1}, \quad (99)$$

$$(\vec{i}_H \cdot \vec{\nabla}) \vec{H} = -\frac{3\mu}{r^4} \sqrt{3 \cos^2 \vartheta + 1} \vec{i}_r - \frac{3\mu}{r^4} \frac{\sin \vartheta \cos \vartheta}{\sqrt{3 \cos^2 \vartheta + 1}} \vec{i}_\vartheta. \quad (100)$$

Forming the product of Eq. (100) with unit vector \vec{i}_H expressed by Eq. (96), we get

$$[\vec{i}_H \times (\vec{i}_H \cdot \vec{\nabla}) \vec{H}] = \vec{i}_\varphi \cdot \frac{3\mu \sin \vartheta}{r^4} \frac{\cos^2 \vartheta + 1}{3 \cos^2 \vartheta + 1}. \quad (101)$$

Further we find

$$[\vec{i}_H \times \vec{\nabla} H] = \vec{i}_\varphi \cdot \frac{3\mu \sin \vartheta}{r^4} \frac{\cos^2 \vartheta + 1}{3 \cos^2 \vartheta + 1}. \quad (102)$$

Using these expressions, we obtain from (92) for the averaged force acting on a charged particle:

$$\vec{F}^{\parallel} = \frac{md\vec{v}^{\parallel}}{dt} = \frac{3W^{\perp}}{r} \cos \vartheta \frac{5 \cos^2 \vartheta + 3}{(3 \cos^2 \vartheta + 1)^{3/2}} \vec{i}_H. \quad (103)$$

For the perpendicular drift velocity, using Eqs. (101) and (102), we obtain

$$\vec{v}^{\perp} = \frac{3r^2 c W}{Ze\mu} \frac{\sin \vartheta (\cos^2 \vartheta + 1)}{(3 \cos^2 \vartheta + 1)^2} \{\sin^2 \alpha + 2 \cos^2 \alpha\} \vec{i}_\varphi \quad (104)$$

or taking into account that $W \sin^2 \alpha = W^{\perp}$

$$\vec{v}^{\perp} = \frac{3r^2 c}{Ze\mu} \frac{\sin \vartheta (\cos^2 \vartheta + 1)}{(3 \cos^2 \vartheta + 1)^2} \{2W - W^{\perp}\} \vec{i}_\varphi. \quad (105)$$

As we see from Eqs. (103) and (105) both the drift velocity and the force acting on particles of a thermal ensemble in magnetic field do not depend on the particle mass. Unlike the other forces, the magnetic force has also a meridional component and this can generate the non-homogeneous chemical composition on the stellar surface correlating with the magnetic field.

Due to fact that, in stellar atmospheres, the frequency of particle impacts is high, the thermal particle velocity distribution can be taken as isotopic. Taking into account that $W = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT$, we can write

$$W^{\perp} = \frac{2}{3} W = kT \quad (106)$$

and

$$W^{\parallel} = \frac{1}{3} W = \frac{1}{2} kT. \quad (107)$$

The equations obtained make evident one of the most peculiar features of magnetic field action on the particles – it depends on particle energies, being essential for high energy particles thus being effective for stellar coronae.

On the polar axis, the magnetic force acting on a particle is

$$\vec{F}^{\parallel} = \frac{3kT}{r} \vec{i}_r.$$

The radius of the main sequence A0 stars is $R \approx 10^{11}$ cm and temperature in atmosphere is $T \approx 10^4$ K. Thus, magnetic force in polar areas is $F^{\parallel} \approx 4 \cdot 10^{-23}$ dyn. The corresponding effective acceleration F^{\parallel}/m for photons equals to 10 cm/s^2 and for electrons it exceeds this value by about 2000 times. For the solar corona, these quantities are two orders of magnitude greater.

The radial component of the stellar magnetic field gives only a small contribution to gravity but the meridional component gives rise to a process of meridional separation of chemical elements.

The drift in the magnetic dipole field does not generate chemical inhomogeneities on the stellar surface because all values of φ are equivalent. The drift motion can be important only in the case of more complicated magnetic field configurations, invoking a change of the chemical element concentrations.

Let us study now modifications which are to be done in the equations of plasma dynamics to describe the drift and diffusion processes for plasma components in the magnetic field. For an adequate formulation of the equations of dynamics for plasma components in the presence of magnetic field, three modifications should be done. First, there appears an additional effective force \vec{F}^{\parallel} directed along the magnetic field lines. Second, the particles drift perpendicularly to the magnetic field. The third and the most important effect is that the effective impact frequency increases essentially in the direction perpendicular to the magnetic field, namely by the factor α^{\perp} , which can be written as (Artsimovich, 1978)

$$\alpha^{\perp} = 1 + (2\pi\nu_{Le}t_{ez})^2, \quad (108)$$

where ν_{Le} is the larmor frequency for electrons,

$$\nu_{Le} = \frac{eH}{2\pi m_e c},$$

and t_{ez} is the mean time interval between electron and ion impacts. Expression (108) has been obtained as a fit formula from two extreme cases of weak and strong magnetic fields. Namely, in the latter case the diffusion coefficient diminishes in the directions perpendicular to magnetic field by $(2\pi\nu_{Le}t_{ez})^2$ times.

In addition, the magnetic field gives rise to the Zeeman effect, i.e. to the splitting and shift of energy levels, thus invoking an additional radiative acceleration of particles. Simultaneously, splitting and shift of energy levels lead to different overlap effects of spectral lines, accompanied by the formation of asymmetric spectral line profiles and thus it can result in stronger light-induced drift effects which are essential in atmospheres of line-rich stars.

Now we study modifications which should be made in the original equation (12) in the presence of magnetic field. Taking into account the contributions of gravitation, electrostatic, magnetostatic and radiation fields, we can write generally

$$\rho_j \vec{a}_j = \rho_j \vec{g}_j + eZ_j n_j \vec{E} + \rho_j a_j^{\parallel} \vec{i}_H + \vec{f}_j. \quad (109)$$

We emphasize once more that all the force fields have different roles for the element separation processes in stellar atmospheres.

The effective impact frequency in the presence of magnetic field depends on direction. In order to take adequately into account this fact, we should make a substitution in the interaction terms of the diffusion and drift equations (12), namely

$$\begin{aligned} \vec{V}_j \rightarrow \vec{V}_j^{\parallel} + \alpha_j \vec{V}_j^{\perp} &= (\vec{i}_H \vec{V}_j) \vec{i}_H + \alpha_j (\vec{V}_j - (\vec{i}_H \vec{V}_j) \vec{i}_H) \\ &= \alpha_j \vec{V}_j + (1 - \alpha_j) (\vec{i}_H \vec{V}_j) \vec{i}_H, \end{aligned} \quad (110)$$

And finally, we should add to the resulting particle velocities \vec{V}_j their drift velocity in the magnetic field

$$\vec{V}_j^H = \vec{V}_j + \vec{v}^\perp. \quad (111)$$

If $\vec{H} = 0$, then all three components of the velocity vector \vec{V}_j are independent and are to be found from independent equations. In the presence of magnetic field the term $(\vec{i}_H \vec{V}_j)$ connects all velocity components with each other. Such a generalized system of equation conserves, however, its linearity and can be solved by the same methods as in the case without magnetic field.

In addition to the diamagnetic moment $\vec{\mu}_L$ due to the Larmor motion given by (92), we should also take into account the paramagnetic moment $\vec{\mu}_A$ of atomic particles. For ions, the substitution $\vec{\mu}_L \rightarrow \vec{\mu}_L - \vec{\mu}_A$ is needed. Taking into account (92) we find

$$\vec{\mu}_L - \vec{\mu}_A = \left(1 - \frac{\mu_A}{\mu_L}\right) \vec{i}_H = \left(1 - \frac{\mu_A H}{kT}\right) \vec{i}_H. \quad (112)$$

When $\mu_A H \ll kT$, the atomic paramagnetic moment can be neglected for ions.

The situation with neutral atoms is opposite - they have only the paramagnetic moment $\vec{\mu}_A$. The force acting on atoms can be obtained from (103) by the substitution $\mu_L = W_\perp/H \rightarrow -\mu_A$, i.e. $W_\perp \rightarrow -H\mu_A$. Explicit expressions for μ_A are beyond the scope of the present paper.

Since the diamagnetic moment of ions and the paramagnetic moment of neutral atoms give forces acting in opposite directions, it is clear that the direction of diffusion depends on the ionization degree in the atmosphere, being directed to the magnetic poles for neutral atoms and to the magnetic equator for ions.

8 COMMENTS AND ESTIMATES ON FORMATION OF CHEMICAL STRATIFICATION IN STELLAR ATMOSPHERES

In the previous sections we restricted ourselves to the chemical element separation with elastic impacts of particles. The approximation is justified when the cross-sections of the elastic collisions of ions exceed considerably those of the inelastic impacts. Typical averaged values of cross-sections for inelastic impacts of singly ionized atoms in stellar atmosphere are $10^{-13} \div 10^{-14}$ cm² while for neutral atoms they are of the order of 10^{-16} cm², and of the same order are the cross-sections of inelastic impacts. Differences of values of inelastic impacts rule the character of light induced drift. The values of elastic collisions for stellar atmospheres range from 10^7 s⁻¹ in uppermost layers of stellar atmospheres to 10^9 s⁻¹ deep in the atmosphere. If the magnetic field strength is about 10^3 G in the upper atmosphere, then according to formula (108) the coefficient of diffusion in perpendicular directions relative to \vec{H} diminishes about 10^6 times, but it is not considerable in deep layers of stellar atmospheres. This factor in connection with meridional circulation directed to the rotation equator of stars leads to an overabundances of highly ionized elements there, i.e. elements with low ionization potentials.

The system of equation obtained can be first solved for dominant buffer gases, i.e. for hydrogen, helium and electrons given by them, obtaining for them separation velocities \bar{V}_ϵ . Therefore the corresponding dominating interaction terms can be used for finding the separation velocities for weak admixture components, ignoring interactions between the admixture components.

For the light-induced drift, the most important factor is the overlap or blending of spectral lines due to which we can obtain for each element an additional drift motion directed in a stellar atmosphere upwards or downwards depending on the details of the overlap. Mostly, however, lighter isotopes drift upwards. For the overlap of isotope spectral lines, there are definite rules – for light elements the spectral lines of higher isotope are shifted bluewards, but for heavy elements beginning from lanthanides the shift is redwards.

The usual radiative flux generated upward acceleration is important for line-rich elements, thus in particular for iron group metals. Due to spectral line shifts and splitting in magnetic fields there appear essential contributions to both the usual radiative acceleration and to the light-induced drift.

The electrostatic field in stellar atmospheres for a pure ionized hydrogen atmosphere is given by $eE = \frac{1}{2}M_H g$ and for a pure doubly ionized helium star it is given by $eE = \frac{1}{2}M_{He} g = 2M_H g$. Thus we can see that in helium-rich atmospheres the hydrogen will be expelled, and the drift of light element isotopes, in particular of hydrogen and helium, are strongly influenced by the electrostatic field.

The phenomena which limit the chemical element separation are stellar wind for B stars, convection for F stars and meridional circulation for rapidly rotating stars. In the case if these limiting factors can only partially compensate the element separation effects, we have an opportunity to obtain valuable additional information on drift velocities of different chemical elements. Additional information about chemical element separation gives also evolutionary effects of CP stars.

It seems that such a valuable information we have in the case of the Sun: in its corona the abundances of elements with the ionization potential less than 10 eV are depleted (4 times) compared with the solar atmosphere and thus there must work effectively a mechanism of element separation which, in the coolest layers of solar atmosphere where $T = 4800$ K, should exceed the velocity $4 \cdot 10^{-4}$ cm/s due to the solar wind. In addition, we mention that the age particles in the solar corona is only about a week.

9 SUMMARY

The problem of the formation of chemical peculiarities in stellar atmosphere occupies essential place in stellar physics. From numerous paper devoted to the problem, it has been concluded that the mechanism of separation of chemical elements in stellar atmospheres is unavoidable due to different forces acting on atoms and ions.

In the present paper we have formulated generalized equations of dynamics for plasma components which can more adequately elucidate and explain the formation

of chemical peculiarities in stellar atmospheres due to the diffusion and drift of chemical elements and their isotopes.

The following results have been obtained:

1. General equations have been derived which describe the generation of chemical peculiarities due to the diffusion and drift of chemical elements in stellar gravitational, electrostatic, magnetic and radiative fields.
2. A formula has been derived for finding electrostatic field that equalizes escape energies for electrons and main ions.
3. It has been demonstrated that in bound-free and free-free transitions the absorbed photons transfer their total momentum to generated ions.
4. General expression for the light-induced drift of chemical elements due to asymmetry of the flux distribution in spectral lines has been found.
5. The equations of diffusion and drift phenomena have been generalized for the case of the presence of magnetic field.
6. Estimations have been given about formation of chemical anomalies and about limiting phenomena – stellar wind and convective turbulence in stellar atmospheres.

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