Contrib. Astron. Obs. Skalnaté Pleso **38**, 445 – 446, (2008)

Formulae for study of light-induced drift diffusion in CP star atmospheres

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Received: November 8, 2007; Accepted: December 21, 2007

Abstract. We present formulae suitable for computing LID acceleration and corresponding diffusive segregation of isotopes in atmospheres of CP stars.

Key words: diffusion – stars: atmospheres – stars: chemically peculiar

Light-induced drift (LID) was first proposed by Atutov and Shalagin (1988) as a phenomenon responsible for separation of isotopes in the atmospheres of CP stars. Thereafter, we have studied evolutionary abundance changes of Hg and its isotopes (see Sapar *et al.*, 2008). Here we present formulae describing diffusion of isotopes in the form best suited to numerical model computations.

Diffusive transfer of radiation holds in the deeper layers of stellar atmospheres, where the monochromatic radiative flux can be described by formula

$$F_\nu = F K_R \frac{dB_\nu}{\kappa_\nu dT}\ , \qquad \text{where}\ \ \frac{1}{K_R} = \int_0^\infty \frac{dB_\nu}{\kappa_\nu dT} d\nu.$$

Here F is the total radiation flux and K_R is the Rosseland opacity integral. In the opacity coefficient κ_ν the overlapping spectral lines of the trace element studied play an important role, and thus $\kappa_\nu = c_\nu + \sum_j \sigma_j W_j(u_\nu)$, where c_ν is the continuous opacity coefficient and σ_j is the transition cross-section per gram. Summation is made over spectral lines j with line profile functions W_j, usually being the Voigt functions with argument given by $u_\nu = (\nu - \nu_j)/\Delta\nu_T$, where $\Delta\nu_T$ is the thermal Doppler width of the spectral line. As we have shown, the effective acceleration of LID due to spectral line j can be expressed in a manner similar to the usual expression of radiative acceleration

$$a_j^L = \frac{\pi\varsigma_j}{c} \int_0^\infty \frac{\partial W_j(u_\nu)}{\partial u_\nu} F_\nu d\nu, \qquad \varsigma_j = q\epsilon\sigma_j, \tag{1}$$

where $q = M v_T c/2h\nu$ and the efficiency of LID is $\epsilon = (C_u - C_l)/(A_u + C_u)$. Here C_u and C_l are the collision rates of particles in the upper and lower states, respectively, and A_u is the probability of spontaneous transitions.

Expression (1) in the regime of diffusive transfer of radiation takes the form

$$a_j^L = \frac{\pi\varsigma_j F K_R}{c} \int_0^\infty \frac{\partial W_j(u_\nu)}{\partial u_\nu} \frac{dB_\nu}{\kappa_\nu dT} d\nu. \tag{2}$$

Both isotopic and hyperfine splitting of spectral lines of all ions should be taken into account to calculate accelerations producing segregation of isotopes.

The equation of continuity for isotope i in a plane-parallel stellar atmosphere has the form $d\rho_i/dt + d(\rho_i V_i)/dr = 0$. The model atmosphere data correspond to standard points, equidistant on a logarithmic scale of the mean optical depth. These points are enumerated as layers n increasing downwards. Treating n as a continuous parameter, we change variables in the equation of continuity. Since $\frac{d}{dr} = \frac{d/dn}{dr/dn}$ and $\rho \frac{dr}{dn} = -\frac{d\mu}{dn}$, where μ is the total column density, we obtain for the radial gradient

$$\frac{d}{dr} = -\gamma \frac{d}{dn}, \quad \text{where } \gamma = \frac{\rho}{d\mu/dn} = \frac{\rho}{\mu d \ln \mu / dn} \, .$$

Denoting the ratio of the current concentration to its initial value as C_i, we can write $\rho_i = \rho_i^0 C_i$ and the equation of continuity reduces to

$$\frac{d \ln C_i}{dt} = \frac{\gamma}{\rho_i} \frac{d(\rho_i V_i)}{dn} \, . \tag{3}$$

Logarithms are used to avoid possible negative values of C_i in the time integration. The diffusion velocity V_i in the presence of a stellar wind can be found from

$$\rho_i V_i = \rho_i (a_i - g) t - \Delta \frac{d\rho_i}{dr} \, , \qquad \frac{d\rho_i}{dr} = -\gamma \frac{d\rho_i}{dn}, \tag{4}$$

where a_i is the sum of the radiative and LID accelerations, t is the mean free flight time of the particles, m is the mean mass of buffer particles and $\Delta = kTt/m$ is the diffusion coefficient of trace particles. Thus from equation (4) we find

$$\frac{V_i}{\gamma \Delta} = \frac{m(a_i - g)}{kT\gamma} + \frac{d \ln(\rho_i^0 C_i)}{dn} \quad \text{and} \quad \frac{V_i}{\gamma \Delta} = \frac{m a_i}{kT\gamma} + \frac{d \ln C_i}{dn}. \tag{5}$$

Here the last expression has been obtained from the first, taking into account that for model stellar atmospheres the condition $mg/kT\gamma = d \ln \rho_i^0 / dn$ is approximately satisfied. The velocity V_i is to be used in the equation of continuity (3), reducing it to a generalized Fokker–Planck equation. Equation (5) can also be used for a crude prediction of final evolutionary concentrations C_i, substituting there $V_i = 0$ if stellar wind is lacking and $V_i = \dot{m}_i \, {\rho_i}^{-1}$ in the case of constant mass loss rate.

As we have seen, for evolutionary computations we need to find derivatives $d \ln \rho / dn$, $d\mu/dn$ and $d\gamma/dn$, which correspond to buffer gases and several derivatives for each isotope of the trace (impurity) particles. These derivatives have been found using the 4th order Lagrange interpolation formulae for equidistant nodes.

Acknowledgements. We are grateful to the Estonian Science Foundation for financial support by grant ETF 6105.

References

Atutov, S.N., Shalagin, A.M.: 1988, *Sov. Astron. Lett.* **14**, 284

Sapar, A., Aret, A., Sapar, L., Poolamäe, R.: 2008, *Contrib. Astron. Obs. Skalnaté Pleso* **38**, 273